Exact solutions and elementary excitations in the XXZ spin chain with unparallel boundary fields

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By using a set of gauge transformations, the exact solutions of the XXZ spin chain with unparallel boundary magnetic fields are derived in the framework of the algebraic Bethe ansatz. In the easy-plane case, we show the elementary excitations are some kind of spinons without definite spin because the $U(1)$ symmetry is broken, while in the easy-axis ferromagnetic case a spiral state is realized in the ground state. The correlation functions, the spin torque as well as the spin voltage for the later case are also derived.

The magnetic structure in a variety of condensed matters has been an important issue in modern physics. In lower dimensions, the situation is especially distinctive and the spins may show a variety of exotic features due to the strong quantum fluctuations. For instance, the elementary excitations in the one-dimensional (1D) spin liquids are doublet kinks [1] rather than triplet spin waves or magnons. Such quanta carry spin 1/2 and are now known as spinons. Furthermore, in spintronics, spins may carry information so that extended capability and performance can be added into the electronic device. [2] Several ways to produce pure spin current (without electric current) have been proposed, such as spiral spin structure as a driving force, [3] spin Hall effect in ferromagnetic metals, [4] and ferromagnetic resonance. [5,6] However, so far no experimental realization of pure spin current has been achieved. The spiral structure is a long range ordered state and therefore could hardly exist in 1D materials such as quantum wires. Some interesting questions one may ask are: Can two unparallel boundary magnetic fields generate a spiral spin state in 1D? How does a spin liquid response to such fields? Since generally these fields break the $U(1)$ symmetry of the system, what is the nature of elementary excitations in 1D spin liquid under such fields?

As proposed in Ref.[3], the spin-supercurrent state may occur in easy-plane ferromagnets. In this work, we show that the spin structure, and consequently, the spin voltage may be controlled by the magnetic anisotropy represented by the boundary magnetic fields. By using a set of local transformations, the initial state for algebraic Bethe ansatz is established and the exact eigen-states of the model Hamiltonian are constructed. In the easy-plane case, the elementary excitations are almost the same as the usual spinons in spectrum but show spiral behavior in the real space and do not carry definite spin because the $U(1)$-symmetry is broken. In the easy-axis ferromagnetic case, the ground state is a spiral state. The magnetization, the correlation functions, the spin torque as well as

the spin voltage in the ground state are derived exactly. The model Hamiltonian we shall consider reads

$$H = \sum_{j=1}^{N-1} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \cos\eta \, \sigma_j^z \sigma_{j+1}^z)$$
$$+ \vec{h}_1 \cdot \vec{\sigma}_1 + \vec{h}_2 \cdot \vec{\sigma}_N, \tag{1}$$

where σ_j^α ($\alpha = x, y, z$) are the Pauli matrices, η is the anisotropic parameter and $\vec{h}_1 = C_1(\sin\tilde{\theta}, \cos\tilde{\theta}, h_1)$ and $\vec{h}_2 = C_2(\sin\theta, \cos\theta, h_2)$ are the boundary fields. If both $\vec{h}_{1,2}$ along the z-direction of the spin space, the system possesses $U(1)$ symmetry, i.e., the z-component of the total spin is conserved and the elementary excitations are the usual spinons. [1]. However, if one of the boundary fields has non-zero transverse component, the $U(1)$ symmetry is broken and the spin structure may be different. We note that the present model has a lot of important applications in several physical systems. It has been providing valuable insights into non-equilibrium behavior and complex dynamics [7,8,9,10] and broken symmetry. [11] Interestingly, this model has been proven to be integrable [12] via the open boundary transfer matrix method. [13] However, the spectrum has not been derived via the conventional Bethe ansatz scheme and leaves as an open problem in the field of integrable models. The difficulty comes from the absence of an obvious initial state or pseudo vacuum (Without the boundary fields, it is the state where all spins polarized along the z-direction), thus the known Bethe ansatz can not be used directly.

In the following text, by using a set of local gauge transformations, we shall construct an adequate initial state with which the eigen-states can be derived via the algebraic Bethe ansatz. To show our procedure clearly, let us review the integrability of the model (1) briefly. The integrability of the XXZ spin chain is associated with the Lax matrix

$$L_{0n}(u) = \frac{1}{2\sin\eta}(\sin(u+\eta) + \sin u)$$
$$+ \frac{1}{2\sin\eta}(\sin(u+\eta) - \sin u)\sigma_0^z\sigma_n^z \qquad (2)$$
$$+ \sigma_0^+\sigma_n^- + \sigma_0^-\sigma_n^+,$$

which satisfies the Yang-Baxter relation [14,15]

$$R_{\alpha\beta}L_{\alpha n}(u)L_{\beta n}(v) = L_{\beta n}(v)L_{\alpha n}(u)R_{\alpha\beta}, \qquad (3)$$

with $R_{\alpha\beta} \equiv L_{\alpha\beta}(u-v)$. The monodromy matrix $T_0(u) = L_{01}(u)\cdots L_{0N}(u)$ also satisfies the Yang-Baxter relation. In addition, as demonstrated by Sklyanin [13], the so called reflection equation

$$R_{\alpha\beta}(u-v)K_\alpha^-(u)R_{\alpha\beta}(u+v)K_\beta^-(v)$$
$$= K_\beta^-(v)R_{\alpha\beta}(u+v)K_\alpha^-(u)R_{\alpha\beta}(u-v) \qquad (4)$$

is also necessary for establishing the open boundary integrable models. Since the double-row monodromy matrix

$$U_0(u) = T_0(u)K_0^-(u)T_0^{-1}(-u) \qquad (5)$$

also satisfies the reflection equation, the transfer matrices $t(u) = tr_0 K_0^+(u)U_0(u)$ with different spectral parameters are mutually commutative, i.e., $[t(u), t(v)] = 0$ as long as the dual reflection matrix $K_0^+(u)$ satisfies the dual reflection equation. [13] Therefore, $t(u)$ serves as the generation function of the conserved quantities. For the XXZ spin chain, the general solution of $K_0^\pm(u)$ read [16,12]

$$K_0^-(u) = c_{00}\cos u - c_{01}\sin u\sigma_0^z$$
$$+ \sin(2u)(\sin\theta\sigma_0^x + \cos\theta\sigma_0^y),$$
$$K_0^+(u) = -\tilde{c}_{00}\cos(u+\eta) - \tilde{c}_{01}\sin(u+\eta)\sigma_0^z \qquad (6)$$
$$+ \sin 2(u+\eta)(\sin\tilde{\theta}\sigma_0^x + \cos\tilde{\theta}\sigma_0^y),$$

where c_{00}, c_{01}, $\tilde{c}_{00}$, $\tilde{c}_{01}$, θ and $\tilde{\theta}$ are arbitrary parameters. The Hamiltonian can be expressed as

$$H = \sin\eta\frac{d}{du}\ln t(u)|_{u=0} - N\cos\eta, \qquad (7)$$

with $C_1 = -2\sin\eta\tilde{c}_{00}^{-1}$, $h_1 = -\tilde{c}_{01}/2$, $C_2 = 2\sin\eta c_{00}^{-1}$ and $h_2 = -c_{01}/2$.

An obvious initial state exists in the conventional algebraic Bethe ansatz scheme, acting on which, the lower left element of the 2×2 matrix $U_0(u)$ is zero. However, in the present case, the initial state is not obvious because K_0^+ and K_0^- are generally not diagonal. To find a proper initial state, as used in solving the eight-vertex model, [15,17,18] we introduce two sets of gauge matrices $M_n(u)$ and $\bar{M}_n(u)$. Under the gauge transformations

$$\bar{L}_{0n}(u|m) = \bar{M}_{n-1+m}^{-1}(u)L_{0n}(u)\bar{M}_{n+m}(u),$$
$$S_{0n}(-u|m) = M_{n+m}^{-1}(-u)L_{0n}^{-1}(-u)M_{n-1+m}(-u), \qquad (8)$$

the lower left elements of $\bar{L}_{0n}$ and its inverse S_{0n} acting on a proper local state is zero. The above requirements define the forms of the gauge matrices as

$$M_n(u) = (X_n(u), Y_n(u)),$$
$$\bar{M}_n(u) = (X_{n+1}(u), Y_{n-1}(u)), \qquad (9)$$

where X_n and Y_n are 1×2 column matrices with upper elements $e^{-iu}x_n$ and $e^{-iu}y_n$, respectively; their lower elements are defined as unity. The constants x_n and y_n read

$$x_n = -ie^{-i\eta(\beta+\gamma+n)}, \ \ y_n = -ie^{-i\eta(\beta-\gamma-n)}, \qquad (10)$$

where β and γ are two parameters determined by the boundary fields. In addition, we define

$$\bar{X}_n(u) = \frac{1}{x_n - y_n}(-e^{iu}, x_n),$$
$$\bar{Y}_n(u) = \frac{1}{x_n - y_n}(e^{iu}, -y_n),$$
$$\tilde{X}_n(u) = \frac{1}{x_n - y_{n-2}}(-e^{iu}, x_n),$$
$$\tilde{Y}_n(u) = \frac{1}{x_{n+2} - y_n}(e^{iu}, -y_n). \qquad (11)$$

In this case, the local vacuum of site n reads

$$|\omega>_n^m = x_{n+m}|\uparrow>_n + |\downarrow>_n. \qquad (12)$$

It is easily to show that the lower left elements of $S_{0n}(-u|m)$ and $\bar{L}_{0n}(u|m)$ acting on this state is zero, while the upper diagonal element of $S_{0n}(-u|m)$ and the lower diagonal element of $\bar{L}_{0n}(u|m)$ shift $|\omega>_n^m$ to $|\omega>_n^{m-1}$, and the lower diagonal element of $S_{0n}(-u|m)$ and the upper diagonal element of $\bar{L}_{0n}(u|m)$ shift $|\omega>_n^m$ to $|\omega>_n^{m+1}$, respectively. Now we can rewrite the transfer matrix as

$$t(u) = tr_0 \tilde{K}_0^+(u|m_0)\tilde{U}_0(u|m_0), \qquad (13)$$

where

$$\tilde{K}_0^+(u|m) = \begin{pmatrix} \tilde{K}_0^+(u|m)_{11} & \tilde{K}_0^+(u|m)_{12} \\ \tilde{K}_0^+(u|m)_{21} & \tilde{K}_0^+(u|m)_{22} \end{pmatrix} =$$
$$\begin{pmatrix} \bar{Y}_m(-u)K^+(u)X_m(u) & \bar{Y}_m(-u)K^+(u)Y_{m-2}(u) \\ \bar{X}_m(-u)K^+(u)X_{m+2}(u) & \bar{X}_m(-u)K^+(u)Y_m(u) \end{pmatrix},$$

and

$$\tilde{U}_0(u|m) = \begin{pmatrix} A_m(u) & B_m(u) \\ C_m(u) & D_m(u) \end{pmatrix} =$$
$$\begin{pmatrix} \tilde{Y}_{m-2}(u)U(u)X_m(-u) & \tilde{Y}_m(u)U(u)Y_m(-u) \\ \tilde{X}_m(u)U(u)X_m(-u) & \tilde{X}_{m+2}(u)U(u)Y_m(-u) \end{pmatrix}. \qquad (14)$$

To ensure $\tilde{K}_0^+(u|m_0)$ being diagonal, we parameterize the left boundary field as $\tilde{c}_{00} + i\tilde{c}_{01} = -2\cos(m_0+\gamma)\eta$, then

$$t(u) = \tilde{K}_0^+(u|m_0)_{11}A_{m_0}(u) + \tilde{K}_0^+(u|m_0)_{22}D_{m_0}(u). \qquad (15)$$

In addition, $C_m(u)$ must annihilate the global initial state $|\Omega>^m = \Pi_\otimes|\omega>_n^m$. This needs $c_{00}+ic_{01} = -2\cos(\theta-\tilde{\theta}+$

$\eta(\gamma + N + m - 1))$, which gives a constraint of $c_{00}, c_{01}, \tilde{c}_{00}$ and $\tilde{c}_{01}$. Under the above condition, $|\Omega >^m$ is a common eigen-state of $A_m(u)$ and $D_m(u)$, and $B_m(u)$ serve as the generating operators of the eigen-states of $t(u)$ as usual. To derive the Bethe states, we need also the commutation relations between A_m, D_m and B_n. With the help of the production relations of $R_{12}(u_1 - u_2)X_{m+2}^1(u_1)X_{m+1}^2(u_2)$ etc. [17] and the reflection equation of $U_0(u)$, after some manipulation we obtain

$$B_m(u_1)B_{m-2}(u_2) = B_m(u_2)B_{m-2}(u_1), \quad (16)$$

$$A_{m+2}(u_1)B_m(u_2) = \frac{\sin(u_1 + u_2)\sin(u_1 - u_2 - \eta)}{\sin(u_1 - u_2)\sin(u_1 + u_2 + \eta)}$$
$$\times B_m(u_2)A_m(u_1)$$
$$- \frac{\sin(2u_2)\sin\eta\sin(u_1 - u_2 - \eta(m + \gamma + 1))}{\sin(u_1 - u_2)\sin\eta(m + \gamma + 1)\sin(2u_2 + \eta)}$$
$$\times B_m(u_1)A_m(u_2)$$
$$- \frac{\sin(-u_1 - u_2 + \eta(m + \gamma))\sin\eta\sin\eta}{\sin\eta(m + \gamma + 1)\sin(u_1 + u_2 + \eta)\sin(2u_2 + \eta)}$$
$$\times B_m(u_1)\tilde{D}_m(u_2), \quad (17)$$

$$\tilde{D}_{m+2}(u_1)B_m(u_2) =$$
$$\frac{\sin(u_1 - u_2 + \eta)\sin(u_1 + u_2 + 2\eta)}{\sin(u_1 - u_2)\sin(u_1 + u_2 + \eta)}$$
$$\times B_m(u_2)\tilde{D}_m(u_1)$$
$$- \frac{\sin(u_1 - u_2 + \eta(m + \gamma + 1))\sin(2u_1 + 2\eta)\sin\eta}{\sin\eta(m + \gamma + 1)\sin(u_1 - u_2)\sin(2u_2 + \eta)}$$
$$\times B_m(u_1)\tilde{D}_m(u_2)$$
$$+ \frac{\sin(2u_2)\sin(u_1 + u_2 + \eta(m + \gamma + 2))}{\sin(u_1 + u_2 + \eta)\sin\eta(m + \gamma + 1)}$$
$$\times \frac{\sin(2u_1 + 2\eta)}{\sin(2u_2 + \eta)}B_m(u_1)A_m(u_2), \quad (18)$$

where

$$\tilde{D}_m(u) = \frac{\sin(2u + \eta)\sin\eta(m + \gamma + 1)}{\sin\eta\sin\eta(m + \gamma)}D_m(u)$$
$$- \frac{\sin(\eta(m + \gamma) + 2u + \eta)}{\sin\eta(m + \gamma)}A_m(u). \quad (19)$$

Assume the eigen-state of the transfer matrix take the form

$$|\Phi > = B_{m_0-2}(u_1)\cdots B_{m_0-2M}(u_M)|\Omega >^m, \quad (20)$$

with $m_0 - 2M = m$. Acting the transfer matrix on this state and exchanging the position of $A_{m_0}(u)$, $\tilde{D}_{m_0}(u)$ and $B_{m_0-2}(u_1)B_{m_0-4}(u_2)\cdots B_{m_0-2M}(u_M)$ by the commutation relations, we obtain two types of terms. One gives the eigen value of $t(u)$ while in the other type one spectral parameter u_j exchanges with u. The latter terms are set to zero and we obtain the Bethe ansatz equations

$$\frac{\sinh^{2N}(\lambda_j - \frac{1}{2}i\eta)}{\sinh^{2N}(\lambda_j + \frac{1}{2}i\eta)} =$$

$$\frac{\sinh(\lambda_j + i\tilde{\alpha}_1 - \frac{1}{2}i\eta)\cosh(\lambda_j + i\tilde{\alpha}_2 - \frac{1}{2}i\eta)}{\sinh(\lambda_j - i\tilde{\alpha}_1 + \frac{1}{2}i\eta)\cosh(\lambda_j - i\tilde{\alpha}_2 + \frac{1}{2}i\eta)}$$
$$\times \frac{\sinh(\lambda_j - i\alpha_1 - \frac{1}{2}i\eta)\cosh(\lambda_j - i\alpha_2 - \frac{1}{2}i\eta)}{\sinh(\lambda_j + i\alpha_1 + \frac{1}{2}i\eta)\cosh(\lambda_j + i\alpha_2 + \frac{1}{2}i\eta)}$$
$$\times \prod_{l \neq j}^M \frac{\sinh(\lambda_j + \lambda_l - i\eta)\sinh(\lambda_j - \lambda_l - i\eta)}{\sinh(\lambda_j + \lambda_l + i\eta)\sinh(\lambda_j - \lambda_l + i\eta)},$$
$$j = 1, 2, \cdots, M. \quad (21)$$

The energy spectrum is given by

$$E = (N - 1)\cos\eta - \frac{\sin\eta\sin(\tilde{\alpha}_1 - \alpha_1)}{\sin\tilde{\alpha}_1\sin\alpha_1}$$
$$- \sum_{j=1}^M \frac{2\sin^2\eta}{\cosh^2\lambda_j - \cos^2(\frac{1}{2}\eta)}, \quad (22)$$

with the parameterizations $c_{00} = -2\sin\alpha_1\cos\alpha_2$, $c_{01} = 2i\cos\alpha_1\sin\alpha_2$, $\tilde{c}_{00} = -2\sin\tilde{\alpha}_1\cos\tilde{\alpha}_2$, $\tilde{c}_{01} = 2i\cos\tilde{\alpha}_1\sin\tilde{\alpha}_2$. Here we have put $u_j \to i\lambda_j - \eta/2$. The Bethe ansatz equations look very similar to that of the usual case. However, there is a restriction on the number of "flipped" spins M. In fact, the two boundary fields are not independent with each other but related with a complex parameter $\gamma + m_0$ and an integer M. That means given a right boundary field, the left boundary is restricted by the right one and M or vise versa. In addition, by exchanging the two fields, we can obtain another set of solutions with $M \to N - M - 1$ and the same Bethe ansatz equations and spectrum. We choose $\tilde{c}_{00} + i\tilde{c}_{01} = -2\cos(m_0 + \gamma)\eta$ and $c_{00} + ic_{01} = -2\cos(\theta - \tilde{\theta} + \eta(m_0 + \gamma - 1))$ for real η (easy-plane case) and even N. In this case, $M = N/2$, $N/2 - 1$. This allows us to study the ground state and the low lying excitation properties simultaneously. In the charge neutral sector ($M = N/2$), the ground state is described by real λ_j which fill the whole real axis in the continuum limit; the excitations are $\lambda - n$-strings with n-holes in the real axis. In the thermodynamic limit $N \to \infty$, the excitation energy reads [19]

$$\epsilon(\lambda_1^h, \cdots \lambda_n^h) = \sum_{j=1}^n \epsilon(\lambda_j^h) = \sum_j^n \frac{\pi\sin\eta}{\eta\cosh\frac{\pi\lambda_j^h}{2\eta}}, \quad (23)$$

i.e., the spectrum takes the same form of the case without the boundary fields, where λ_j^h is the position of j-th hole in the spectral space and $\epsilon(\lambda_j^h)$ is the energy of a single spinon. [19] The other type of excitations can be constructed by considering the process of $M = N/2$ to $M = N/2 - 1$, i.e., a two-hole state based on the ground state. The excitation energy simply reads

$$\epsilon(\lambda_1^h, \lambda_2^h) = \epsilon(\lambda_1^h) + \epsilon(\lambda_2^h), \quad (24)$$

where $\lambda_{1,2}^h$ are the positions of the two holes in the spectral space. This state corresponds to the spin triplet excitation in the usual case. The above discussion shows that

the elementary excitations are not changed by the boundary fields in spectrum, i.e, they are also described by spinons formally (additive spectrum). Since the spinons are some kind of topological excitations, it is rather strange that the broken symmetry does not alter the spectrum. However, the spin structure in the real space is indeed changed by the boundary fields. The initial state is in fact a spiral state with the spins rotate site by site in the $x-y$ plane. The "spin-wave" generators $B_m(u)$ do not "flip" the spins on different sites to an uniform direction. Therefore the spins have winding behavior around the z-direction.

For the easy-axis ferromagnetic case, we put $\eta = \pi + i\chi$ with χ real and study the $\tilde{c}_{00} + i\tilde{c}_{01} = -2\cos((\pi + i\chi)(m_0+\gamma))$, and $c_{00}+ic_{01} = -2\cos(\theta-\tilde{\theta}+(\pi+i\chi)(m_0+\gamma+N-1))$ case. The ground state is described by the reference state $|\Omega>^{m_0}$. Several quantities in the ground state can be derived easily. For instance, the z-direction magnetization is

$$< M_z > = \frac{1}{2} + O(N^{-1}), \tag{25}$$

implying that the boundary fields change the magnetization only in order of $1/N$. The correlation functions can also be derived as

$$
\begin{aligned}
< \sigma_n^z \sigma_m^z > &= \tanh f(n) \tanh f(m), \\
< \sigma_n^x \sigma_m^x > &= \frac{(-1)^{n+m}\cos^2(g)}{\cosh f(n) \cosh f(m)}, \\
< \sigma_n^y \sigma_m^y > &= \frac{(-1)^{n+m}\sin^2(g)}{\cosh f(n) \cosh f(m)},
\end{aligned}
\tag{26}
$$

with $f(n) = (Re(m_0 + \gamma) + n - 1)\chi + \pi Im(m_0 + \gamma)$ and $g = \pi Re(m_0 + \gamma) - \chi Im(m_0 + \gamma) - \tilde{\theta}$. Obviously, the z-z correlation function tends to 1 when both m and n tend to infinity, indicating the long-range order of the ground state. Define the quantum spin torque operator as $\vec{S}_n = \frac{1}{4}\vec{\sigma}_n \times \vec{\sigma}_{n+1}$. The expectation value of this quantity in the ground state reads

$$
\begin{aligned}
< S_n^x > &= (-1)^n \frac{\sin(g) \sinh[f(n) + \chi/2] \cosh(\chi/2)}{2 \cosh f(n+1) \cosh f(n)}, \\
< S_n^y > &= (-1)^{n+1} \frac{\cos(g) \sinh[f(n) + \chi/2] \cosh(\chi/2)}{2 \cosh f(n+1) \cosh f(n)}, \\
< S_n^z > &= 0.
\end{aligned}
$$

The spin voltage can also be calculated as $V_s^\alpha = \sum_{n=1}^{N-1} < S_n^\alpha >$, $\alpha = x, y, z$.

In conclusion, we develop a method to derive the eigenstates of the XXZ spin chain with unparallel boundary fields. For some special choices of the boundary fields, the ground state and the low lying excitations for the easy-plane case are obtained exactly. The spectrum of the excitations is of the same form as that of the usual spinons without definite charge (spin), suggesting that there exist some hidden symmetries in the system, an interesting question needs to be further explored. For the easy-axis ferromagnetic case, the ground state is a pure spiral state which possesses non-zero spin torque in the x-y plane. This provides a possible scheme to generate driving force for spin current in quasi 1D systems.

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